Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Tuesday 15 October 2024

Morning (Time: 1 hour 30 minutes)

Paper reference **WMA12/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Pure Mathematics P2

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 11 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over

Pearson

1. A continuous curve has equation $y = f(x)$.

A table of values of x and y for $y = f(x)$ is shown below.

Interval btwn two x-coords = h of trapezium

x	0.5	1.75	3	4.25	5.5
y	3.479	6.101	7.448	6.823	5.182

y_1 multiplied by 2 as all are shared by two trapezia y_6

Using the trapezium rule with all the values of y in the given table,

(a) find an estimate for

$$\int_{0.5}^{5.5} f(x)\,dx$$

giving your answer to one decimal place.

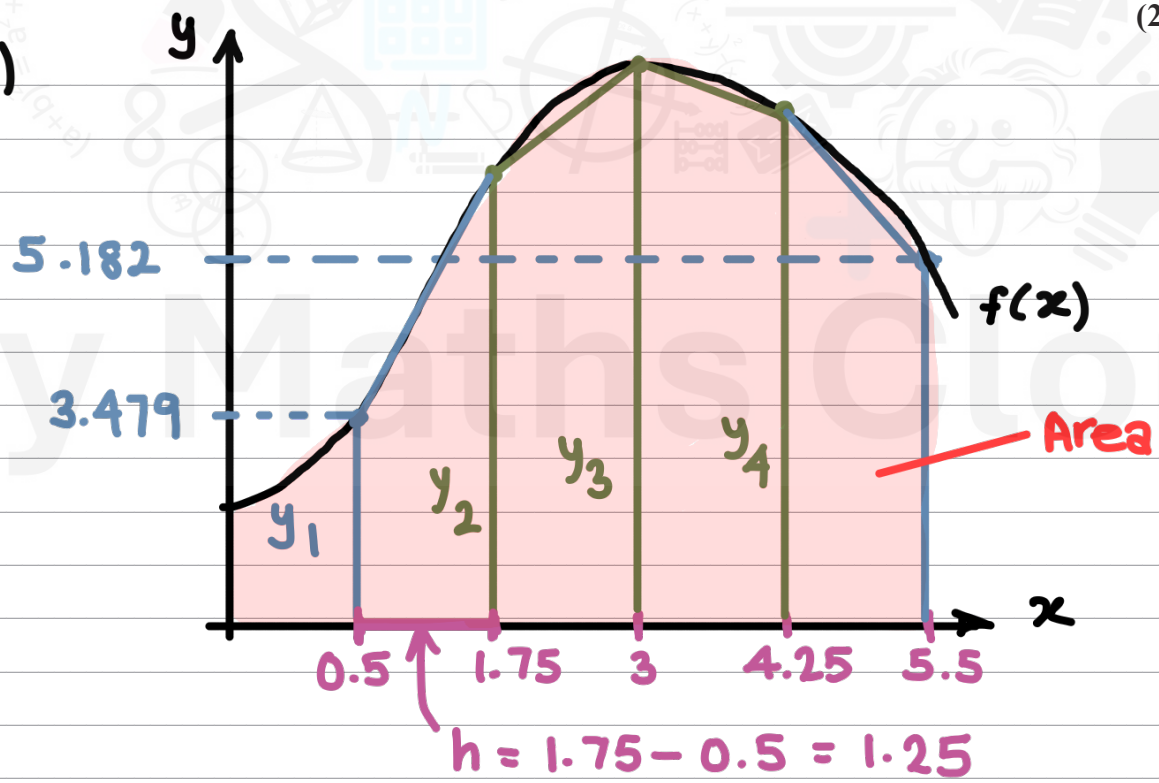
(3)

(b) Using your answer to part (a) and making your method clear, estimate

$$\int_{0.5}^{5.5} \left(f(x) + 4x\right) dx$$

(2)

(a)



(Formula is given in formula booklet. The graph is only a visual representation. DO NOT DRAW A GRAPH IN THE EXAM)

Question 1 continued

$$A = \frac{1}{2}h\left[y_1 + 2(y_2 + y_3 + y_4) + y_6\right]$$

$$A = \frac{1}{2}(1.25)\left[3.479 + 2(6.101 + 7.448 + 6.823) + 5.182\right] = 30.8781\ldots$$

So: $\int_{0.5}^{5.5} f(x)\,dx \approx 30.8781\ldots$

$= 30.9$ (1 d.p)

(b)

$$\boxed{\text{use rule: } \int f(x) + g(x)\,dx = \int f(x)\,dx + \int g(x)\,dx}$$

$$\int_{0.5}^{5.5} f(x) + 4x\,dx = \int_{0.5}^{5.5} f(x)\,dx + \int_{0.5}^{5.5} 4x\,dx$$

$$\int_{0.5}^{5.5} f(x)\,dx + \int_{0.5}^{5.5} 4x\,dx$$

$$= 30.9 + \left[\frac{4x^{1+1}}{1+1}\right]_{0.5}^{5.5}$$

REMEMBER!

$$\boxed{\int x^n\,dx = \frac{x^{n+1}}{n+1}}$$

$$= 30.9 + \left[2x^2\right]_{0.5}^{5.5}$$

$$= 30.9 + \left(2(5.5)^2 - 2(0.5)^2\right)$$

$$= 30.9 + 60 = 90.9$$

(Total for Question 1 is 5 marks)

2. A sequence of numbers $u_1, u_2, u_3, \ldots$ is defined by

$$u_1 = 7$$

$$\boxed{u_{n+1} = (-1)^n u_n + k}$$

recursive sequence (plugging in a term on the right gives the next term on the left)

where k is a constant.

(a) Show that $u_5 = 7$ **(3)**

end

terms : $u_1, u_2, u_3 \ldots$

Given that $\sum_{r=1}^{4} u_r = 30$

sum

starting value

(b) find the value of k. **(2)**

(c) Hence find the value of $\sum_{r=1}^{150} u_r$ **(2)**

(a) This is a recursive sequence :

$u_{n+1} = (-1)^n u_n + k$ → notice how $n+1$ is 1 more than n

If $u_1 = 7$:

$u_{1+1} = (-1)^1 (7) + k$
$= (-1)(7) + k)$
$u_2 = -7 + k$

$u_{2+1} = (-1)^2 (-7+k) + k$
$= (1)(-7+k) + k$
$u_3 = -7 + 2k$

$u_{3+1} = (-1)^3 (-7+2k) + k$
$= 7 - 2k + k$
$u_4 = 7 - k$

$u_{4+1} = (-1)^4 (7-k) + k$
$= 1(7-k) + k$
$u_5 = 7$

Question 2 continued

(b) $\sum_{r=1}^{4} u_r = 30$ means start the sequence at $r=1$ and sum all the terms up to $r=4$

$u_1 + u_2 + u_3 + u_4 = 30$

$7 + -7+k + -7+2k + 7-k = 30$

$2k = 30$

$k = 15$

(c) $\sum_{r=1}^{150} u_r = u_1 + u_2 + u_3 + u_4 + \dots u_{150}$

We have seen in (a) that the sequence repeats itself every 4 terms ($u_1 = u_5$).

$\underbrace{7 + -7+k + -7+2k + 7-k}\,+\underbrace{7 + -7+k + -7+2k + 7-k}\,+\dots$

So: $\frac{150}{4\text{ terms}} = 37 + 2$

remainder, 2 more terms

37 repeats (a_1 to a_4)

We know that $u_1 + u_2 + u_3 + u_4 = 30$.

So: $\sum_{r=1}^{150} u_r = 37(30) + 7 + (-7+15)$

$= 1125$

(Total for Question 2 is 7 marks)

3.

$$f(x) = 2x^3 - x^2 + Ax + B$$

where A and B are integers.

$= (x - (-3))$

Given that when $f(x)$ is divided by $(x + 3)$ the remainder is 55

(a) show that

$$3A - B = -118$$

(2)

Given also that $(2x - 5)$ is a factor of $f(x)$, → no remainder when dividing

(b) find the value of A and the value of B.

(3)

(c) Hence find the quotient when $f(x)$ is divided by $(x - 7)$

(2)

(a) USE : REMAINDER THEOREM :

if $\dfrac{f(x)}{(x-k)}$ = quotient + remainder :

then $f(k)$ = remainder

If $\dfrac{f(x)}{(x-(-3))}$ = quotient + 55

then $f(-3) = 55$:

$2(-3)^3 - (-3)^2 + A(-3) + B = 55$

$-54 - 9 - 3A + B = 55$

$3A - B = -54 - 9 - 55$

$3A - B = -118$

(b) FACTOR THEOREM : (no remainder)

If $\dfrac{f(x)}{(ax-k)}$ = quotient + 0 , then $f\left(\dfrac{k}{a}\right) = 0$

If $\dfrac{f(x)}{(2x-5)} = 0$, then $f\left(\dfrac{5}{2}\right) = 0$:

Question 3 continued

$$2\left(\frac{5}{2}\right)^3 - \left(\frac{5}{2}\right)^2 + A\left(\frac{5}{2}\right) + B = 0$$

$$\frac{125}{4} - \frac{25}{4} + \frac{5}{2}A + B = 0$$

$$2 \times \left(\frac{5}{2}A + B\right) = (-25) \times 2$$

$$5A + 2B = -50$$

NOW WE HAVE TWO EQ. :

① $3A - B = -118$
② $5A + 2B = 50$

SOLVE SIMULTANEOUSLY

① elimination :

$[3A - B = -118] \times 2$
$5A + 2B = -50$

$$\oplus \quad \begin{aligned} 6A - 2B &= -236 \\ 5A + 2B &= -50 \\ \hline 11A &= -286 \end{aligned}$$

$A = -26$

$3(-26) - B = -118$
$B = 40$

② Substitution :

$3A - B = -118$
$B = 3A + 118$

$5A + 2B = -50$
$5A + 2(3A + 118) = -50$
$5A + 6A + 236 = -50$
$11A = -286$
$A = -26$

$B = 3(-26) + 118$
$B = 40$

SO : $A = -26$
$B = 40$

Question 3 continued

(c) $f(x) = 2x^3 - x^2 - 26x + 40$

Use algebraic division to find quotient :

$$x - 7 \overline{\big) 2x^3 - x^2 - 26x + 40}$$

(1) Divide first term of dividend by first term of divisor : $\frac{2x^3}{x} = 2x^2 \rightarrow$ first term of quotient

$$\begin{array}{r|l} & 2x^2 \\ x - 7 & 2x^3 - x^2 - 26x + 40 \end{array}$$

(2) Multiply out and write under :

$$\begin{array}{r|l} & 2x^2 \\ x - 7 & 2x^3 - x^2 - 26x + 40 \\ & 2x^3 - 14x^2 \end{array}$$

(3) Subtract then bring down next term :

$$\begin{array}{r|l} & 2x^2 \\ x - 7 & 2x^3 - x^2 - 26x + 40 \\ & -(2x^3 - 14x^2) \downarrow \\ \hline & 13x^2 - 26x \end{array}$$

(4) Divide first term of expression by first term of divisor : $\frac{13x^2}{x} = 13x \rightarrow$ next term of quotient

$$\begin{array}{r|l} & 2x^2 + 13x \\ x - 7 & 2x^3 - x^2 - 26x + 40 \\ & -(2x^3 - 14x^2) \\ \hline & 13x^2 - 26x \end{array}$$

Question 3 continued

(8) Multiply out and write under:

$$
\begin{array}{r|l}
 & 2x^2 + 13x \\
\hline
x - 7 & 2x^3 - x^2 - 26x + 40 \\
 & -(2x^3 - 14x^2) \\
\hline
 & 13x^2 - 26x \\
 & 13x^2 - 91x \\
\hline
\end{array}
$$

(6) Subtract then bring down next term:

$$
\begin{array}{r|l}
 & 2x^2 + 13x \\
\hline
x - 7 & 2x^3 - x^2 - 26x + 40 \\
 & -(2x^3 - 14x^2) \\
\hline
 & 13x^2 - 26x \\
 & -(13x^2 - 91x) \\
\hline
 & 65x + 40
\end{array}
$$

(7) Divide first term of expression by first term of divisor: $\frac{65x}{x} = 65$ → next term of quotient

$$
\begin{array}{r|l}
 & 2x^2 + 13x + 65 \\
\hline
x - 7 & 2x^3 - x^2 - 26x + 40 \\
 & -(2x^3 - 14x^2) \\
\hline
 & 13x^2 - 26x \\
 & -(13x^2 - 91x) \\
\hline
 & 65x + 40
\end{array}
$$

(8) Multiply out and write under:

$$
\begin{array}{r|l}
 & 2x^2 + 13x + 65 \\
\hline
x - 7 & 2x^3 - x^2 - 26x + 40 \\
 & -(2x^3 - 14x^2) \\
\hline
 & 13x^2 - 26x \\
 & -(13x^2 - 91x) \\
\hline
 & 65x + 40 \\
 & 65x - 455
\end{array}
$$

(Total for Question 3 is 7 marks)

Question 3 continued

(9) Subtract

$$
\begin{array}{r|l}
 & 2x^2 + 13x + 65 \\
\hline
x - 7 & 2x^3 - x^2 - 26x + 40 \\
 & -(2x^3 - 14x^2) \\
\hline
 & 13x^2 - 26x \\
 & -(13x^2 - 91x) \\
\hline
 & 65x + 40 \\
 & -(65x - 455) \\
\hline
 & 495
\end{array}
$$

so : $\dfrac{f(x)}{x-7} = \underbrace{2x^2 + 13x + 65}_{\text{quotient.}} + \underbrace{495}_{\text{remainder!}}$

quotient $= 2x^2 + 13x + 65$

TOTAL FOR PAPER IS 75 MARKS

4. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

The curve C has equation

$$y = 4x^{\frac{1}{2}} + 9x^{-\frac{1}{2}} + 3 \qquad x > 0$$

don't ignore this!

(a) Find $\frac{dy}{dx}$ giving each term in simplest form. **(2)**

$dy/dx = 0$

(b) Hence find the x coordinate of the stationary point of C. **(2)**

(c) (i) Find $\frac{d^2y}{dx^2}$ giving each term in simplest form.

(ii) Hence determine the nature of the stationary point of C, giving a reason for your answer. **(2)**

(d) State the range of values of x for which y is decreasing. **(1)**

(a)

DIFFERENTIATION

$$y = ax^n \longrightarrow \frac{dy}{dx} = nax^{n-1}$$

$$y = 4x^{1/2} + 9x^{-1/2} + 3$$

$$\frac{dy}{dx} = \left(\frac{1}{2}\right)4x^{1/2-1} + \left(-\frac{1}{2}\right)9x^{-1/2-1}$$

$$= 2x^{-1/2} - \frac{9}{2}x^{-3/2}$$

(b) $\frac{dy}{dx} = 0$ at stationary point

$$2x^{-1/2} - \frac{9}{2}x^{-3/2} = 0$$

$$x^{3/2} \times \left(2x^{-1/2} - \frac{9}{2}x^{-3/2}\right) = (0) \times x^{3/2}$$

$$2x^{-1/2+3/2} - \frac{9}{2}x^{-3/2+3/2} = 0 \rightarrow \left(a^b \times a^c = a^{b+c}\right)$$

$$2x - \frac{9}{2} = 0 \longrightarrow \boxed{x = \frac{9}{4}}$$

Question 4 continued

(c)(i) $\frac{dy}{dx} = 2x^{-1/2} - \frac{9}{2}x^{-3/2}$

$\frac{d^2y}{dx^2} = \left(\frac{-1}{2}\right)2x^{-1/2-1} - \left(-\frac{3}{2}\right)\frac{9}{2}x^{-3/2-1}$

$\frac{d^2y}{dx^2} = -x^{-3/2} + \frac{27}{4}x^{-5/2}$

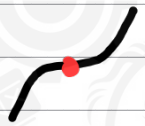
(ii) at $x = 9/4$:

$\frac{d^2y}{dx^2} = -\left(\frac{9}{4}\right)^{-3/2} + \frac{27}{4}\left(\frac{9}{4}\right)^{-5/2} = \frac{16}{27}$

RECALL :

if $\frac{d^2y}{dx^2} > 0 \rightarrow$ minimum point

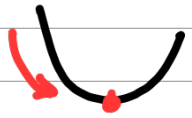
$\frac{d^2y}{dx^2} = 0 \rightarrow$ point of inflection



$\frac{d^2y}{dx^2} < 0 \rightarrow$ maximum point

$\frac{16}{27} > 0$ so point is a minimum :)

(d) $\frac{9}{4}$ is a minimum :



y is decreasing BEFORE $9/4$:

$0 < x < 9/4$

because range in question is for $x > 0$!!

(Total for Question 4 is 7 marks)

5. (a) Find, in terms of a, the first 3 terms, in ascending powers of x, of the binomial expansion of

$$(2 + ax)^6$$

where a is a non-zero constant. Give each term in simplest form.

(3)

$$f(x) = \left(3 + \frac{1}{x}\right)^2 (2 + ax)^6$$

Given that the constant term in the expansion of $f(x)$ is 576

(b) find the value of a.

(4)

(a) Binomial expansion formula :

$$\boxed{(a+bx)^n = {}^nC_0 a^n (bx)^0 + {}^nC_1 a^{n-1}(bx)^1 + {}^nC_2 a^{n-2}(bx)^2 + \dots}$$

$${}^nC_r = \frac{n!}{(n-r)!\,r!}$$

n : power of binomial

r : (position of term) − 1

OR USE CHOOSE FUNCTION ON CALCULATOR!

$$(2+ax)^6 = {}^6C_0\, 2^6 (ax)^0 + {}^6C_1\, 2^5 (ax)^1 + {}^6C_2\, 2^4 (ax)^2 + \dots$$

${}^6C_0 = 1$

${}^6C_1 = 6$

${}^6C_2 = \frac{6!}{(6-2)!\,2!} = 15$

NOTES :-
- the first and last ${}^nC_r$ are always = 1
- ${}^nC_1$ is always equal to n

$$(2+ax)^6 = 1(64)(1) + 6(32)(ax) + 15(16)(a^2x^2) + \dots$$

$$= 64 + 192ax + 240a^2x^2$$

Question 5 continued

expand using quick method

$$f(x) = \left(3 + \frac{1}{x}\right)^2 (2 + ax)^6$$

expansion estimated in (a).

$$f(x) = \left(9 + \frac{6}{x} + \frac{1}{x^2}\right)\left(64 + 192ax + 240a^2x^2\right)$$

After multiplying, the constant terms include all terms where :

- no x is present in either multiplied value :

$$9 \times 64 = 576$$

- the x value cancels out :

$$\frac{6}{x} \times 192ax = 1152a$$

- the x^2 value cancels out :

$$\frac{1}{x^2} \times 240a^2x^2 = 240a^2$$

So : $\cancel{576} + 1152a + 240a^2 = \cancel{576}$

$$a(240a + 1152) = 0$$

$[a = 0]$ or $240a + 1152 = 0$

reject, a is non-zero!

$$a = \frac{-1152}{240} = -\frac{24}{5}$$

FINAL ANSWER!

(Total for Question 5 is 7 marks)

P 7 6 1 8 7 R A 0 1 3 3 2

6. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

Using the laws of logarithms, solve

$$\log_4(12 - 2x) = 2 + 2\log_4(x + 1)$$

(5)

Recall rules of logarithms ;

$\log_a(b) + \log_a(c) = \log_a(b \times c)$

$\log_a(b) - \log_a(c) = \log_a\left(\frac{b}{c}\right)$

$\log a^b = b\log a$

$\log_a b = c \rightarrow a^c = b$

$\log_4(12-2x) = 2 + 2\log_4(x+1)$

$\log_4(12-2x) - 2\log_4(x+1) = 2$

$\log_4(12-2x) - \log_4(x+1)^2 = 2$

$\log_4\left(\frac{12-2x}{(x+1)^2}\right) = 2$

$(x+1)^2 \times \left(\frac{12-2x}{(x+1)^2}\right) = (4^2) \times (x+1)^2$

$12 - 2x = 16(x^2 + 2x + 1)$

$12 - 2x = 16x^2 + 32x + 16$

$\frac{16x^2}{2} + \frac{34x}{2} + \frac{4}{2} = 0$

$8x^2 + 17x + 2 = 0$

$(8x + 1)(x + 2) = 0$

FINAL ANSWER $x = -1/8$ or $x = -2 \rightarrow$ REJECT ; $\log_4(-2+1) = \log_4(-1)$, log of -ve value has no real solution!

Question 6 continued

ALTERNATIVE SOLUTION :

$$\log_4(12-2x) = 2 + 2\log_4(x+1)$$

$$2\log_4(x+1) - \log_4(12-2x) = -2$$

$$\log_4(x+1)^2 - \log_4(12-2x) = -2$$

$$\log_4\left(\frac{(x+1)^2}{12-2x}\right) = -2$$

$$(12-2x) \times \left(\frac{(x+1)^2}{12-2x}\right) = \left((4)^{-2}\right) \times (12-2x)$$

$$x^2+2x+1 = \frac{1}{16}(12-2x)$$

$$x^2+2x+1 = \frac{12}{16} - \frac{2}{16}x$$

$$\left(x^2 + \frac{17}{8}x + \frac{1}{4} = 0\right) \times 8$$

$$8x^2 + 17x + 2 = 0$$

$$(8x+1)(x+2) = 0$$

$x = -1/8$ or $x = -2$ → REJECT ; $\log_4(-2+1) = \log_4(-1)$, log of -ve value has no real solution!

FINAL ANSWER

(Total for Question 6 is 5 marks)

P 7 6 1 8 7 R A 0 1 5 3 2

7. Jem pays money into a savings scheme, A, over a period of 300 months.

Jem pays £20 (a_1) into scheme A in month 1, £20.50 (a_2) in month 2, £21 (u_3) in month 3 and so on, so that the amounts Jem pays each month form an arithmetic sequence.

(a) Show that Jem pays £69.50 into scheme A in month 100

(1)

partial sum S_n $n=300$

(b) Find the **total** amount that Jem pays into scheme A over the period of 300 months.

(2)

Kim pays money into a different savings scheme, B, over the same period of 300 months.

In a model, the amounts Kim pays into scheme B increase by the same percentage (r) each month, so that the amounts Kim pays each month form a geometric sequence.

Given that Kim pays

- £20 (a_1) into scheme B in month 1
- £250 (a_n) into scheme B in month 300

(c) use the model to calculate, to the nearest £10, the difference between the total amount paid into scheme A and the total amount paid into scheme B over the period of 300 months.

(3)

(a) $a_1, a_2, a_3, \ldots, a_{100}$

£20, £20.50, £21, ..., £??

+0.50 +0.5

For arithmetic series:

$a_n = a + d(n-1)$

$a_{100} = 20 + 0.5(100-1) = £69.50$

(b) use partial sum formula:

$S_n = \frac{n}{2}[2a + d(n-1)]$

$= \frac{300}{2}[2(20) + 0.5(300-1)] = £28\,425$

Question 7 continued

OR: Use partial sum formula:

$$S_n = \frac{n}{2}\left[a_1 + a_n\right]$$

$$a_{300} = 20 + 0.5(300-1) = 169.5$$

$$S_{300} = \frac{300}{2}\left[20 + 169.5\right] = £28\,425$$

(c) ① Find r

$$a_n = ar^{n-1}$$
$$250 = 20r^{300-1}$$

$$\sqrt[299]{r^{299}} = \sqrt[299]{\frac{25}{2}} \rightarrow r = \sqrt[299]{\frac{25}{2}}$$

use the exact value for calculations!

② Use partial sum formula:

$$S_n = \frac{a_1(1-r^n)}{(1-r)}$$

$$S_{300} = \frac{20\left(1-\left(\sqrt[299]{\frac{25}{2}}\right)^{300}\right)}{\left(1-\sqrt[299]{\frac{25}{2}}\right)}$$

$$S_{300} = 27\,362.948\ldots$$
$$= £27\,362$$

difference $= £28\,425 - £27\,362 = 1063$
$= £1060$
(nearest ten)

Question 7 continued

Question 7 continued

(Total for Question 7 is 6 marks)

8. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

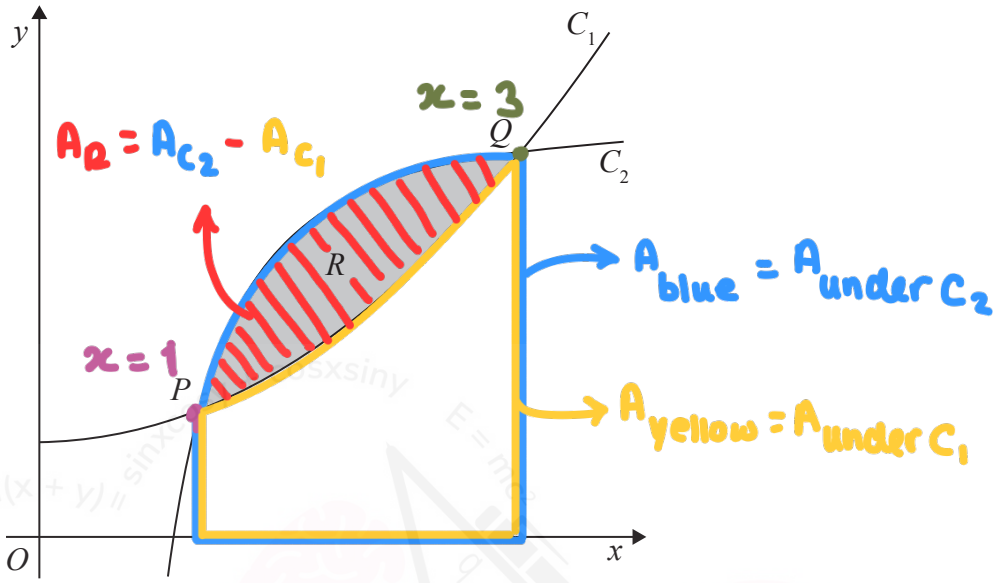


Figure 1

Figure 1 shows a sketch of part of the curve C_1 with equation

$$y = x^2 + 3 \qquad x > 0$$

and part of the curve C_2 with equation

$$y = 13 - \frac{9}{x^2} \qquad x > 0$$

The curves C_1 and C_2 intersect at the points P and Q as shown in Figure 1.

(a) Use algebra to find the x coordinate of P and the x coordinate of Q.

(4)

The finite region R, shown shaded in Figure 1, is bounded by C_1 and C_2

(b) Use algebraic integration to find the exact area of R.

(4)

(a) P and Q lie on C_1 and C_2, so:

at x_P and x_Q, $y_{C_1} = y_{C_2}$

$x^2 \times (x^2 + 3) = \left(13 - \frac{9}{x^2}\right) \times x^2$

$x^4 + 3x^2 = 13x^2 - 9$

Question 8 continued

$$x^4 - 10x^2 + 9 = 0$$

$$(x^2)^2 - 10(x^2)^1 + 9 = 0$$ → can be factorised like a quadratic equation!

$$(x^2 - 1)(x^2 - 9) = 0$$

$$x^2 - 1 = 0 \quad \text{or} \quad x^2 - 9 = 0$$

$$x = 1 \quad \text{or} \quad x = 3$$

$$x_P = 1$$

$$x_Q = 3$$

(b) $A_R = A_{C2} - A_{C1}$

$$A_R = \int_1^3 13 - \frac{9}{x^2}\,dx - \int_1^3 x^2 + 3\,dx$$

COMBINE USING RULE:

$$\boxed{\int f(x)\,dx \pm \int g(x)\,dx = \int f(x) \pm g(x)\,dx}$$

$$A_R = \int_1^3 13 - \frac{9}{x^2} - (x^2 + 3)\,dx$$

$$A_R = \int_1^3 13 - 9x^{-2} - x^2 - 3\,dx$$

$$A_R = \int_1^3 10 - 9x^{-2} - x^2\,dx$$

REMEMBER!

$$\boxed{\int x^n\,dx = \frac{x^{n+1}}{n+1}}$$

Question 8 continued

$$A_R = \left[\frac{10x^{0+1}}{0+1} - \frac{9x^{-2+1}}{-2+1} - \frac{x^{2+1}}{2+1} \right]_1^3$$

$$A_R = \left[10x + \frac{9}{x} - \frac{x^3}{3} \right]_1^3$$

$$A_R = \left[10(3) + \frac{9}{3} - \frac{3^3}{3} \right] - \left[10(1) + \frac{9}{1} - \frac{1^3}{3} \right]$$

$$A_R = (30 + 3 - 9) - (10 + 9 - 1/3)$$

$$A_R = 24 - \frac{56}{3}$$

$$A_R = \frac{16}{3}$$

9. **In this question you must show detailed reasoning.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that the equation

$$2\tan\theta = 3\cos\theta$$

can be written as

$$3\sin^2\theta + 2\sin\theta - 3 = 0$$

(3)

(b) Hence solve, for $-\pi < x < \pi$, the equation

$$2\tan\left(2x + \frac{\pi}{3}\right) = 3\cos\left(2x + \frac{\pi}{3}\right)$$

giving your answers to 3 significant figures.

(4)

(a) $2\tan\theta = 3\cos\theta$ $\quad \tan\theta = \frac{\sin\theta}{\cos\theta}$

$\cos\theta \times \left(2\frac{\sin\theta}{\cos\theta}\right) = (3\cos\theta) \times \cos\theta$

$2\sin\theta = 3\cos^2\theta$

Use identity: $\sin^2\theta + \cos^2\theta = 1$
$\cos^2\theta = 1 - \sin^2\theta$

$2\sin\theta = 3(1 - \sin^2\theta)$

$2\sin\theta = 3 - 3\sin^2\theta$

$\therefore 3\sin^2\theta + 2\sin\theta - 3 = 0$

(b) $3\sin^2\left(2x + \frac{\pi}{3}\right) + 2\sin\left(2x + \frac{\pi}{3}\right) - 3 = 0$

let $\sin^2(2x + \pi/3) = x$

$3x^2 + 2x - 3 = 0 \rightarrow$ FACTORISE!

Question 9 continued

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-2 \pm \sqrt{2^2 - 4(3)(-3)}}{2(3)} \rightarrow x = \frac{-1 \pm \sqrt{10}}{3}$$

$$\sin\left(2x + \frac{\pi}{3}\right) = \frac{-1 - \sqrt{10}}{3} \times$$ REJECT ; range of $y = \sin\theta$ is $-1 < \theta < 1$

$$\sin\left(2x + \frac{\pi}{3}\right) = \frac{-1 + \sqrt{10}}{3} \rightarrow = 0.72\ldots$$ so accept !

NOW, SOLVE :

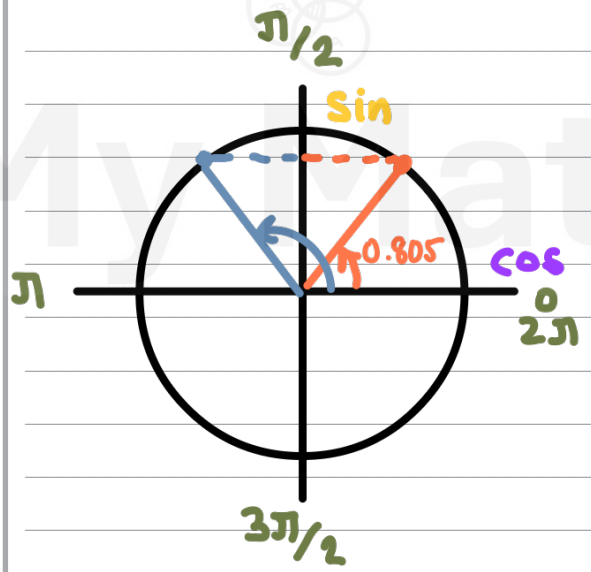
$$\sin\left(2x + \frac{\pi}{3}\right) = \frac{-1 + \sqrt{10}}{3}, \quad \left[2(-\pi) + \frac{\pi}{3} < 2x + \frac{\pi}{3} < 2(\pi) + \frac{\pi}{3}\right]$$

domain !

ON CALCULATOR :

$$\left(2x + \frac{\pi}{3}\right) = \sin^{-1}\left(\frac{-1 + \sqrt{10}}{3}\right) = 0.8049$$

ON UNIT CIRCLE :



SOL1: $2x + \pi/3 = 0.8049$

SOL2: $2x + \pi/3 = \pi - 0.8049$
$= 2.337$

these two solutions are only within the domain of $0 < 2x + \pi/3 < 2\pi$.

(Total for Question 9 is 7 marks)

Question 9 continued

Since our domain is actually :

$$-\frac{5}{3}\pi < 2x + \pi/3 < \frac{7\pi}{3}$$

SOL 3 : $2x + \pi/3 = 2.337 - 2\pi$
$= -3.946$

SOL 4 : $2x + \pi/3 = 0.8049 + 2\pi$
$= 7.088$

$2x + \frac{\pi}{3} = 0.8049, -3.949, 2.337, 7.088$

Subtract $\pi/3$, divide by 2 ;

$x = -0.121, -2.50, 0.645, 3.02$ (3 s.f.)

10. The circle C has equation

$$x^2 + y^2 + 4x - 30y + 209 = 0$$

(a) Find

(i) the coordinates of the centre of C,

(ii) the exact value of the radius of C.

(3)

The line L has equation $y = mx + 1$, where m is a constant.

Given that L is the tangent to C at the point P,

(b) show that

$$2m^2 - 7m - 22 = 0$$

(3)

(c) Hence find the possible pairs of coordinates of P.

(4)

(a) Standard form of equation of a circle:

$$(x - x_{centre})^2 + (y - y_{centre})^2 = r^2$$

↳ rewrite given eq. into standard form:

$$x^2 + 4x + y^2 - 30y + 209 = 0$$

(1) complete the square for terms with x:

$$x^2 + 4x + y^2 - 30y = -209$$

↳ add $\left(\frac{b}{2}\right)^2$ to both sides:

$$x^2 + 4x + \left(\frac{(-4)}{2}\right)^2 + y^2 - 30y = -209 + \left(\frac{(-4)}{2}\right)^2$$

$$x^2 + 4x + 4 + y^2 - 30y = -209 + 4$$

↳ factorise

Question 10 continued

$(x+2)^2 + y^2 - 30y = -205$

(3) complete the square for terms with y :

$(x+2)^2 + y^2 - 30y = -205$

↳ add $\left(\frac{b}{2}\right)^2$ to both sides :

$(x+2)^2 + y^2 - 30y + \left(\frac{-30}{2}\right)^2 = -205 + \left(\frac{-30}{2}\right)^2$

$(x+2)^2 + y^2 - 30y + 225 = -205 + 225$

↳ factorise

$(x+2)^2 + (y-15)^2 = 20$

so : $(x \underset{-(-2)}{+} 2)^2 + (y-15)^2 = 20$

(i) coords of centre : $(-2, 5)$

(ii) radius → $r^2 = 20$, $r = \sqrt{20}$

(b) At point P, $y_C = y_L$

↳ substitute $y = mx+1$ into y_C :

$(x+2)^2 + (mx+1-15)^2 = 20$

$(x+2)^2 + (mx-14)^2 = 20$

$x^2 + 4x + 4 + m^2x^2 - 28mx + 196 = 20$

$x^2 + m^2x^2 + 4x - 28mx + 180 = 0$

Question 10 continued

$$\underbrace{(1+m^2)}_{a}x^2 + \underbrace{(4-28m)}_{b}x + \underbrace{180}_{c} = 0$$

Since line L and circle C only touch at ONE POINT, we can use the discriminant to form an equation with m alone.

REMEMBER:

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

discriminant

If discriminant >0, then eq. has two solutions
If discriminant $=0$, then eq. has one solution
If discriminant <0, then eq. has no solutions

$$b^2 - 4ac = 0$$

$$(4-28m)^2 - 4(1+m^2)(180) = 0$$

$$16 - 224m + 784m^2 - 4(180 + 180m^2) = 0$$

$$16 - 224m + 784m^2 - 720 - 720m^2 = 0$$

$$\frac{64m^2}{32} - \frac{224m}{32} - \frac{704}{32} = 0$$

$$\therefore 2m^2 - 7m - 22 = 0$$

(c) $2m^2 - 7m - 22 = 0 \rightarrow$ FACTORISE!

$$(2m-11)(m+2) = 0$$

$m = 11/2$ or $m = -2$

Question 10 continued

① WHEN $m = 11/2$:

$$x^2 + \left(\frac{11}{2}\right)^2 x^2 + 4x - 28\left(\frac{11}{2}\right)x + 180 = 0$$

$$x^2 + \frac{121}{4}x^2 + 4x - 154x + 180 = 0$$

$$4 \times \left(\frac{125}{4}x^2 - 150x + 180\right) = (0) \times 4$$

$$125x^2 - 600x + 720 = 0 \rightarrow \text{FACTORISE}$$

$$(5x - 12)^2 = 0$$

$$x = \frac{12}{5}$$

$$y = \frac{11}{2}\left(\frac{12}{5}\right) + 1 = \frac{71}{5}$$

② WHEN $m = -2$:

$$x^2 + (-2)^2 x^2 + 4x - 28(-2)x + 180 = 0$$

$$x^2 + 4x^2 + 4x + 56x + 180 = 0$$

$$\frac{5x^2}{5} + \frac{60x}{5} + \frac{180}{5} = 0$$

$$x^2 + 12x + 36 = 0 \rightarrow \text{FACTORISE}$$

$$(x + 6)^2 = 0$$

$$x = -6$$

$$y = \frac{11}{2}(-6) + 1 = 13$$

possible coordinates: $\left(\frac{12}{5}, \frac{71}{5}\right)$ or $(-6, 13)$

(Total for Question 10 is 10 marks)

11. (i) Prove by counter example that the statement

"If n is a prime number then $3^n + 2$ is also a prime number."

is false.

(2)

(ii) Use proof by exhaustion to prove that if m is an integer that is **not** divisible by 3, then

$$m^2 - 1$$

is divisible by 3

(4)

(i) Give any example that disproves the statement :

ex : $n = 5$: $3^5 + 2 = 245$
$= 5 \times 7 \times 7$ NOT PRIME

$n = 7$: $3^7 + 2 = 2189$
$= 11 \times 199$ NOT PRIME

$n = 11$: $3^{11} + 2 = 177\ 149$
$= 7 \times 25\ 307$ NOT PRIME

(ii) If m is divisible by 3, $m = 3k$

If m is NOT divisible by 3 :

$m = 3k+1$ $\qquad$ $m = 3k+2$

WHEN $m = 3k+1$:

$(3k+1)^2 - 1$
$= 9k^2 + 6k + 1 - 1$
$= 9k^2 + 6k$
$= 3(3k^2 + 2k)$

WHEN $m = 3k+2$

$(3k+2)^2 - 1$
$= 9k^2 + 12k + 4 - 1$
$= 9k^2 + 12k + 3$
$= 3(3k^2 + 4 + 1)$

Question 11 continued

$3(3k^2+2k)$ and $3(3k^2+4+1)$ are both multiples of 3, so:

m^2-1 must be divisible by 3 when m is not divisible by 3.